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General Transportability – Synthesizing Observations and Experiments from Heterogeneous Domains

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Abstract

The process of transporting and synthesizing experimental findings from heterogeneous data collections to construct causal explanations is arguably one of the most central and challenging problems in modern data science. This problem has been studied in the causal inference literature under the rubric of causal effect identifiability and transportability (Bareinboim and Pearl 2016). In this paper, we investigate a general version of this challenge where the goal is to learn conditional causal effects from an arbitrary combination of datasets collected under different conditions, observational or experimental, and from heterogeneous populations. Specifically, we introduce a unified graphical criterion that characterizes the conditions under which conditional causal effects can be uniquely determined from the disparate data collections. We further develop an efficient, sound, and complete algorithm that outputs an expression for the conditional effect whenever it exists, which synthesizes the available causal knowledge and empirical evidence; if the algorithm is unable to find a formula, then such synthesis is provably impossible, unless further parametric assumptions are made. Finally, we prove that do-calculus (Pearl 1995) is complete for this task, i.e., the inexistence of a do-calculus derivation implies the impossibility of constructing the targeted causal explanation.

1 Introduction

In the empirical sciences, experiments are almost invariably performed with the intent of being used elsewhere (e.g., outside the laboratory), where the conditions are likely to be different. This practice is based on the premise that, owing to certain commonalities between the source and target environments, causal claims will be valid even where experiments have never been carried out. In biology, for example, many experiments performed on *Bonobos* are not designed due to an inherent interest in this particular species, but because of their similarity to *Homo Sapiens*, and the hope that the experimental findings would be robust, and transportable across species. The capability of generalizing causal knowledge plays a critical role in machine learning as well; an intelligent system is trained in one environment — where it is allowed to perform interventions — with the goal of operating more efficiently and surgically in a deployment site, despite their structural differences (Pearl 2000; Bareinboim and Pearl 2016).

One natural question that arises in these settings is what makes scientists believe that experimental studies conducted in one species could, at least in principle, be used to make causal claims about another different one? Also, how could AI engineers expect, or perhaps hope, that an intelligent system trained in one environment would operate successfully when deployed in a different location? The key observation leveraged in these cases is that, while there might exist glaring differences in the source and target domains, some mechanisms are shared across domains, and owed to their invariances, they would act as anchors, allowing knowledge to be transported and causal learning to take place eventually (Pearl 2000; Spirtes, Glymour, and Scheines 2001; Bareinboim and Pearl 2016; Pearl and Mackenzie 2018).

The fields of machine learning and artificial intelligence provide the theoretical underpinnings to reason about causal mechanisms so as to tackle the challenge of synthesizing experimental findings in a principled and systematic way. In particular, we build on the framework of structural causal models (SCMs) (Pearl 2000) to formalize this setting and systematically leverage the invariant features of the underlying data-generating model. An increasingly large class of problems regarding the generalizability of experimental findings across domains has been studied in the last decades within the SCM framework. For instance, the problem of *identifiability* of causal effects has been investigated, which is concerned with the conditions under which the causal effect of a treatment variable (or set) X on an outcome variable (or set) Y, usually written as $P(Y|do(X))$, can be determined from the combination of the observational distribution and qualitative understanding about the domain encoded in the form of a causal diagram. A criterion known as the *backdoor* has been introduced in (Pearl 1993), which provided a formal, graphical justification for when causal effects can be identified by the adjustment formula, and then estimated by propensity score methods. There exist a number of other conditions developed to solve this problem (Galles and Pearl 1995; Pearl and Robins 1995; Kuroki and Miyakawa 1999; Halpern 2000; Spirtes, Glymour, and Scheines 2001). Pearl introduced *do*-calculus as a general algebraic solution to this problem, which is applicable when observational and/or experimental distributions are available (Pearl 1995). Based on this machinery, more general graphical and algorithmic identifiability conditions were derived, which culminated in complete characterizations (Tian 2002; Tian and Pearl 2002; Shpitser and Pearl 2006b; Huang and Valtorta 2006; Bareinboim and Pearl 2012a; Lee, Correa, and Bareinboim 2019).

More recently, the problem of generalizing causal distributions across heterogeneous domains[1] has been studied under the rubric of *transportability* (Pearl and Bareinboim 2011). Early work in transportability considered whether the experiments coming from a *source* domain can be leveraged to answer a query in a *target* domain, despite the two domains differing in some of their underlying mechanisms (Bareinboim and Pearl 2012b). This setting was then generalized to allow multiple source domains, different set of manipulable variables per domain, or both (Bareinboim and Pearl 2014). Transportability has been used in more applied settings, for example, (Westreich and Edwards 2015; Westreich et al. 2017; Lesko et al. 2017; Keiding and Louis 2018; Zhou et al. 2018). See also discussions in (Pearl 2015; Pearl and Mackenzie 2018; Pearl and Bareinboim 2019).

Despite the many advances achieved in the transportability literature throughout the past decade, each work addressed some of the following specific dimensions: 1. (conditional) a causal query can be of a *conditional* interventional probability instead of only marginal; 2. (specification) available data can be of an *arbitrary collection* of observational and experimental distributions instead of a restricted class (e.g., all combinations of experiments); and 3. (heterogeneity) the data can come from a number of *heterogeneous* domains. While it lies beyond the scope of this paper to provide a survey of this body of literature, for the sake of clarity, we provide a short summary of the relationship between its main settings in Appendix A.1.

We will account for these three aspects simultaneously, and ultimately provide a solution to the most general version of transportability. Cohesively combining the disparate machinery (e.g., concepts, conditions, algorithms) developed for these different instances of the transportability problem turns out to be a non-trivial task since they capture different aspects of the problem, operating at distinct levels of abstraction. The main goal of this paper, technically speaking, will be to put these results together under a general, unifying umbrella. More specifically, our contributions are as follows: (1) We derive a necessary and sufficient graphical criterion for determining whether conditional interventional distributions (including unconditional and observational distributions) in a target domain can be uniquely determined from a set of observational and experimental distributions spread throughout heterogeneous domains; (2) We develop a sound and complete algorithm for this problem. (3) We then prove that *do*-calculus (Pearl 1995) is complete for the task of general transportability.

[1]Some general tasks found in the sciences can be seen as instances of transportability theory, including external validity (Campbell and Stanley 1963; Manski 2007), meta-analysis (Hedges and Olkin 1985), quasi-experiment (Shadish, Cook, and Campbell 2002), or heterogeneity (Morgan and Winship 2007).

1.1 Preliminaries

We use uppercase letters for variables and lowercase for the corresponding values. We denote by $\mathfrak{X}_V$ the state space of V where $v \in \mathfrak{X}_V$. A bold letter represents a set. Calligraphic letters are for mathematical structures such as graphs and models. We use familial notation for relationships among vertices in a graph: $Pa(\cdot)$, $An(\cdot)$, and $De(\cdot)$ represent parents, ancestors, and descendants of variables (including its argument as well). In this paper, we are interested in graphs, induced from a SCM (to be defined formally), with both directed and bidirected edges. The *root set* of a graph is a set of vertices with no outgoing edge. Given a graph $\mathcal{G}$, we use $\mathbf{V}$ to represent the set of vertices in $\mathcal{G}$ in the current scope if no ambiguity arises. Otherwise, we denote by $\mathbf{V}(\mathcal{G}')$ the set of observed variables in $\mathcal{G}'$. We denote by $\mathcal{G}[\mathbf{W}]$ a subgraph induced on $\mathcal{G}$ by $\mathbf{W}$, which consists of $\mathbf{W}$ and edges among them. We define $\mathcal{G} \setminus \mathbf{Z}$ as $\mathcal{G}[\mathbf{V} \setminus \mathbf{Z}]$. We denote by $\mathcal{G}_{\overline{\mathbf{X}}}$ and $\mathcal{G}_{\underline{\mathbf{X}}}$ edge-subgraphs of $\mathcal{G}$ with incoming edges onto $\mathbf{X}$ and outgoing edges from $\mathbf{X}$, respectively, removed. We adopt set-related symbols for graphs, e.g., $\mathcal{G}' \subseteq \mathcal{G}$ denotes $\mathcal{G}'$ being a subgraph of $\mathcal{G}$, or $\mathcal{T} \cup \mathcal{H}$ stands for the union of two graphs $\mathcal{T}$ and $\mathcal{H}$.

As mentioned, we use the language of SCMs (Pearl 2000, Ch. 7) as our basic semantical framework, which allows us to represent observational and interventional distributions as well as different domains. Formally, a tuple $\langle \mathbf{U}, \mathbf{V}, \mathbf{F}, P(\mathbf{U}) \rangle$ defines a SCM $\mathcal{M}$ where i) $\mathbf{U}$ is a set of unobserved variables; ii) $\mathbf{V}$ is a set of observed variables; iii) $\mathbf{F}$ is a set of deterministic functions $\{f_V\}_{V \in \mathbf{V}}$ for observed variables, e.g., $v \leftarrow f_V(\mathbf{pa}_V, \mathbf{u}_V)$ where $\mathbf{PA}_V \subseteq \mathbf{V} \setminus \{V\}$ and $\mathbf{U}_V \subseteq \mathbf{U}$; and iv) $P(\mathbf{U})$ is a joint probability distribution over $\mathbf{U}$. Intervening on $\mathbf{X}$ by fixing it to $\mathbf{x}$, denoted by $do(\mathbf{X} = \mathbf{x}) = do(\mathbf{x})$, in $\mathcal{M}$ creates a submodel $\mathcal{M}_{\mathbf{x}} = \langle \mathbf{U}, \mathbf{V}, \mathbf{F}_{\mathbf{x}}, P(\mathbf{U}) \rangle$ where $\mathbf{F}_{\mathbf{x}}$ is $\mathbf{F}$ with f_X replaced by a constant x for every $X \in \mathbf{X}$. The submodel $\mathcal{M}_{\mathbf{x}}$ induces an interventional distribution $P_{\mathbf{x}}$, which is also denoted by $P(\cdot \mid do(\mathbf{x}))$. A SCM induces a causal diagram where its vertices correspond to $\mathbf{V}$, directed edges represent functional relationships as specified in $\mathbf{F}$, and each of bidirected edges portrays the existence of an unobserved confounder (UC) between the two vertices pointed by the edge. We will make extensive use of the *do*-calculus, which is a set of three rules that allow one to reason about invariances across observational and experimental distributions. For all the proofs and appendices, please refer to the full technical report (Lee, Correa, and Bareinboim 2020).

2 Towards General Transportability

In this section, we introduce some basic results needed to formalize and solve the problem of general transportability.

In this work, we consider the set of heterogeneous domains (i.e., environments, studies, or populations) $\Pi = \{\pi^1, \pi^2, \ldots, \pi^n\}$, where each domain associates with a SCM compatible with a common causal diagram $\mathcal{G}$. We fix

π^1 as a *target* domain in which we are interested in answering a causal query, and the others are considered *source* domains. Through out this paper, let $* = 1$ to emphasize the target domain, e.g., π^* or P^*. The distributions associated with π^i under $do(\mathbf{x})$ will be denoted by $P^i_{\mathbf{x}}$. Following the construction in (Bareinboim and Pearl 2012b), we formally characterize structural heterogeneity across domains:

Definition 1 (Domain Discrepancy)**.** Let π^a and π^b be domains associated, respectively, with SCMs $\mathcal{M}^a$ and $\mathcal{M}^b$ conforming to a causal diagram $\mathcal{G}$. We denote by $\Delta^{a,b} \subseteq \mathbf{V}$ a set of variables such that, for every $V \in \Delta^{a,b}$, there might exist a discrepancy; either $f^a_V \neq f^b_V$ or $P^a(\mathbf{U}_V) \neq P^b(\mathbf{U}_V)$.

Further, the differences between the target and each of the source domains is represented in $\mathcal{G}$:

Definition 2 (Selection Diagram)**.** Given a collection of domain discrepancies $\mathbf{\Delta} = \{\Delta^{*,i}\}^n_{i=1}$ with regard to $\mathcal{G} = \langle \mathbf{V}, \mathbf{E} \rangle$, let $\mathbf{S} = \{S_V \mid \exists^n_{i=1} V \in \Delta^{*,i}\}$ be *selection variables*. Then, a *selection diagram* $\mathcal{G}^{\mathbf{\Delta}}$ is defined as a graph $\langle \mathbf{V} \cup \mathbf{S}, \mathbf{E} \cup \{S_V \rightarrow V\}_{S_V \in \mathbf{S}} \rangle$.

We shorten $\Delta^{*,i}$ as Δ^i to represent the differences between the target and each source domain. We denote domain-specific selection variables by $\mathbf{S}^i = \{S_V\}_{V \in \Delta^i}$, and the rest by $\mathbf{S}^{-i} = \mathbf{S} \setminus \mathbf{S}^i$. Selection variables work like switches selecting the domain of interest. The state space of $S_V \in \mathbf{S}$ is $\{1\} \cup \{i \mid V \in \Delta^i \in \mathbf{\Delta}\}$. Therefore, a selection diagram can be viewed as the causal diagram for a unifying SCM[2] representing heterogeneous SCMs where $P_{\mathbf{x}}(\mathbf{y} \mid \mathbf{w}, \mathbf{s}^i = \mathbf{i}, \mathbf{s}^{-i} = \mathbf{1}) = P^i_{\mathbf{x}}(\mathbf{y} \mid \mathbf{w})$.

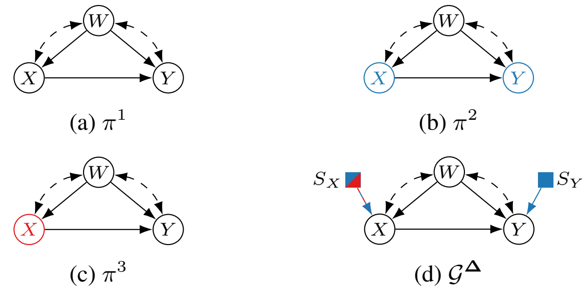
For example, we illustrate in Figs. 1a to 1c a common causal graph $\mathcal{G}$ among three domains with different colors to highlight the discrepancies between the target and source domains. This corresponds to $\mathbf{\Delta} = \{\emptyset, \{X, Y\}, \{X\}\}$, which entails the selection diagram $\mathcal{G}^{\mathbf{\Delta}}$ in Fig. 1d. We are now ready to define the most general transportability instance that will be investigated in this paper, namely:

Definition 3 (g-Transportability)**.** Let $\mathcal{G}^{\mathbf{\Delta}}$ be a selection diagram relative to domains $\Pi = \{\pi^i\}^n_{i=1}$ with a target domain π^*. Let $\mathbb{Z} = \{\mathbb{Z}^i\}^n_{i=1}$ be a specification of available experiments, where $\mathbb{Z}^i$ is the collection of sets of variables for π^i in which experiments on each set of variables $\mathbf{Z} \in \mathbb{Z}^i$ can be conducted. Given disjoint sets of variables $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{W}$, the conditional causal effect $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is said to be g-transportable given $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$ if $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is uniquely computable from $\mathbb{P}^{\Pi}_{\mathbb{Z}} = \{P^i_{\mathbf{z}} \mid \mathbf{z} \in \mathfrak{X}_{\mathbf{Z}}, \mathbf{Z} \in \mathbb{Z}^i \in \mathbb{Z}\}$ in any collection of models that induce $\mathcal{G}^{\mathbf{\Delta}}$.

This problem can be seen as asking about the existence of a functor g that outputs a universal formula given $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$, which takes $\mathbb{P}^{\Pi}_{\mathbb{Z}}$ and returns $P^*_{\mathbf{x}}(\mathbf{y} \mid \mathbf{w})$, i.e., $\exists_g P^*_{\mathbf{x}}(\mathbf{y} \mid \mathbf{w}) = g(\mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z})(\mathbb{P}^{\Pi}_{\mathbb{Z}})$. Again, considering the selection diagram in Fig. 1d with $\mathbb{Z} = \{\emptyset, \{\{Y\}\}, \{\{X\}\}\}$, one can show that

$$P^*_x(y|w) = \frac{P^*_x(y,w)}{P^*_x(w)} = \frac{P^3_x(y,w)}{P^*_x(w)} = \frac{P^3_x(y,w)}{P^2_y(w)} \qquad (1)$$

Specifically, note that the first equality follows from the definition of conditional probability, the second one is due to the irrelevance of the different X mechanisms between π^* and π^3 under $do(x)$, and the last one is based on Rule 3 (removing $do(x)$ and adding $do(y)$) together with W being indifferent to the disparities on f_X and f_Y between π^* and π^2. The following lemma provides a declarative way to determine whether a query $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is g-transportable given $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$ based on the selection diagram.



(a) π^1 (b) π^2 (c) π^3 (d) $\mathcal{G}^{\mathbf{\Delta}}$

Figure 1: Causal graphs colored to depict the discrepancies between (a) a target domain; (b,c) two source domains where $\mathbf{\Delta} = \{\emptyset, \{X, Y\}, \{X\}\}$, which induces $\mathbf{S} = \{S_X, S_Y\}$, where $\mathbf{S}^2 = \{S_X, S_Y\}$; $\mathbf{S}^3 = \{S_X\}$ and (d) a selection diagram $\mathcal{G}^{\mathbf{\Delta}}$.

Lemma 1. *A causal effect $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is g-transportable with respect to $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$ if the expression $P_{\mathbf{x}}(\mathbf{y}|\mathbf{w}, \mathbf{S})$ is reducible to an expression in which every term of the form $P_{\mathbf{a}}(\mathbf{b}|\mathbf{c}, \mathbf{S}')$ satisfies $(\mathbf{S} \setminus \mathbf{S}' \perp\!\!\!\perp \mathbf{B} \mid \mathbf{C})$ in $\mathcal{G}^{\mathbf{\Delta}} \setminus \mathbf{A}$, $\mathbf{S}^i \cap \mathbf{S}' = \emptyset$, and $\mathbf{A} \in \mathbb{Z}^i$ for some domain $\pi^i \in \Pi$.*

Proof. The condition implies that $P_{\mathbf{x}}(\mathbf{y}|\mathbf{w}, \mathbf{s}{=}\mathbf{1})$ can be written as an expression with terms, e.g. $P_{\mathbf{a}}(\mathbf{b}|\mathbf{c}, \mathbf{s}'{=}\mathbf{1})$, and further entails that $P_{\mathbf{a}}(\mathbf{b}|\mathbf{c}, \mathbf{s}'{=}\mathbf{1}) = P_{\mathbf{a}}(\mathbf{b}|\mathbf{c}, \mathbf{s}^{-i}{=}\mathbf{1}, \mathbf{s}^i{=}\mathbf{i}) = P^i_{\mathbf{a}}(\mathbf{b}|\mathbf{c})$ for any π^i such that $\mathbf{S}^i \cap \mathbf{S}' = \emptyset$. Since $P^i_{\mathbf{a}} \in \mathbb{P}^{\Pi}_{\mathbb{Z}}$, the expression uniquely computes $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ with $\mathbb{P}^{\Pi}_{\mathbb{Z}}$. □

The previous example in Eq. (1) on Fig. 1 can be rewritten by explicitly employing the selection variables to articulate the applications of *do*-calculus and the axioms of probability:

$$P_x(y|w, \mathbf{S}) = \frac{P_x(y, w|\mathbf{S})}{P_x(w|\mathbf{S})} = \frac{P_x(y, w|S_Y)}{P_x(w)} = \frac{P^3_x(y,w)}{P^2_y(w)}$$

For instance, $P_x(y, w|S_Y) = P^3_x(y,w)$ due to $\{S_Y\} \subseteq \mathbf{S}^{-3} = \{S_X, S_Y\} \setminus \{S_X\}$. We next characterize non-g-transportability of a conditional causal effect:

Lemma 2. *A causal effect $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is not g-transportable with respect to $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$, if there exist two SCMs compatible with $\mathcal{G}^{\mathbf{\Delta}}$ where both agree on $\mathbb{P}^{\Pi}_{\mathbb{Z}}$ while disagreeing on $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$.*

[2] One can construct a SCM $\mathcal{M} = \langle \cup_i \mathbf{U}^i, \mathbf{V} \cup \mathbf{S}, \mathbf{F}, \prod_i P^i(\mathbf{U}^i) \rangle$ where $\mathbf{F}$ is the same as the one in $\mathcal{M}^1$ except $X \in \mathbf{V}$ such that $S_X \in \mathbf{S}$. For such a variable X, adopt $X = f^{S_X}_X(\mathbf{PA}_X, \mathbf{U}^{S_X}_X)$, which selects the given domain's function as specified by S_X.

Proof. Having two different values for the query $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ rules out the existence of a valid function mapping from $\langle \mathcal{G}^{\boldsymbol{\Delta}}, \mathbb{Z}\rangle$ to the conditional causal effect. □

The conditional causal effect $P^*_x(y|w)$ shown in Fig. 1 would not be g-transportable if π^3 associates with an observational distribution without an experiment on X, i.e., $\mathbb{Z}^3 = \{\emptyset\}$; or if its mechanism on W disagrees with π^*, i.e., $\Delta^3 = \{W\}$. We will provide a graphical criterion for the non-g-transportability of a query in Sec. 3 based on Lemma 2, and devise a sound and complete algorithm for the problem of g-transportability in Sec. 4 grounded on Lemma 1 and the results in Sec. 3.

3 A Graphical Criterion for Non-g-transportability

We present a graphical criterion which can tell whether a conditional causal effect is not g-transportable. We start by examining the case of an unconditional causal effect (Sec. 3.1). These results will be leveraged to investigate conditional effects (Sec. 3.2).

3.1 Non-g-transportability of an Unconditional Interventional Distribution

We investigate a graphical characterization of non-g-transportability of an unconditional causal effect given $\langle \mathcal{G}^{\boldsymbol{\Delta}}, \mathbb{Z}\rangle$. We formally introduce essential notions devised in the identifiability literature (Tian and Pearl 2002; Shpitser and Pearl 2006b) with slight revisions. A subgraph of $\mathcal{G}$ is called a *C-component* (Tian 2002; Tian and Pearl 2002) if its bidirected edges form a spanning tree over all vertices in the subgraph. A graph $\mathcal{G}$ can be decomposed into a set of maximal C-components. We denote by $\mathcal{C}(\mathcal{G})$ the decomposition of $\mathbf{V}$ with respect to maximal C-components. An $\mathbf{R}$-rooted *C-forest* is a C-component whose root set is $\mathbf{R}$ and edges are minimal such that every vertex other than $\mathbf{R}$ has one child and bidirected arcs form a spanning tree. A pair of C-forests with an inclusive relationship, often denoted by $\langle \mathcal{F}, \mathcal{F}'\rangle$ such that $\mathcal{F}' \subseteq \mathcal{F}$, sharing the same roots is called a *hedge*. If there exists an $\mathbf{R}$-rooted hedge $\langle \mathcal{F}, \mathcal{F}'\rangle$ in $\mathcal{G}$ with $\mathbf{R} \subseteq An(\mathbf{Y})_{\mathcal{G}\setminus\mathbf{X}}$, $\mathbf{X} \cap \mathcal{F} \neq \emptyset$, and $\mathbf{X} \cap \mathcal{F}' = \emptyset$, then we say that $\langle \mathcal{F}, \mathcal{F}'\rangle$ is formed for $P^*_{\mathbf{x}}(\mathbf{y})$, which implies that the same effect is not identifiable in $\mathcal{G}$ from P (Shpitser and Pearl 2006b). For example, $\mathcal{F}_a$ in Fig. 2b is a $\{Y_1, R, Y_2\}$-rooted C-forest. The subgraph made of this root-set alone is also a $\{Y_1, R, Y_2\}$-rooted C-forest. That is, the pair $\langle \mathcal{F}_a, \mathcal{F}_a[\{Y_1, R, Y_2\}]\rangle$ is a hedge, which is formed for $P^*_{x_1}(y_1, y_2)$ in $\mathcal{G}$ (but not for $P^*_{x_1}(y_1)$).

Thicket is a graphical structure that precludes the non-identifiability of $P^*_{\mathbf{x}}(\mathbf{y})$ with $\langle \mathcal{G}^{\{\emptyset\}}, \{\mathbb{Z}^*\}\rangle$ (i.e., a single domain with an arbitrary collection of experiments) (Lee, Correa, and Bareinboim 2019). We introduce the notion of *s-thicket*, a generalization of a thicket to a heterogeneous setting by taking selection variables into account:

Definition 4 (s-Thicket)**.** Given $\langle \mathcal{G}^{\boldsymbol{\Delta}}, \mathbb{Z}\rangle$, an s-thicket $\mathcal{T}$ is a minimal non-empty $\mathbf{R}$-rooted C-component of $\mathcal{G}$ such that for each $\mathbf{Z} \in \mathbb{Z}^i \in \mathbb{Z}$, either (a) $\Delta^i \cap \mathbf{R} \neq \emptyset$, (b) $\mathbf{Z} \cap \mathbf{R} \neq \emptyset$, or (c) there exists $\mathcal{F} \subseteq \mathcal{T} \setminus \mathbf{Z}$ where $\langle \mathcal{F}, \mathcal{T}[\mathbf{R}]\rangle$ is a hedge. If $\mathbf{R} \subseteq An(\mathbf{Y})_{\mathcal{G}\setminus\mathbf{X}}$ and every *hedgelet* of the hedges intersects with $\mathbf{X}$, we say an s-thicket $\mathcal{T}$ is formed for $P^*_{\mathbf{x}}(\mathbf{y})$ in $\mathcal{G}^{\boldsymbol{\Delta}}$ with respect to $\mathbb{Z}$.

Definition 5 (hedgelet decomposition)**.** The hedgelet decomposition $\mathbb{H}(\langle \mathcal{F}, \mathcal{F}'\rangle)$ of a hedge $\langle \mathcal{F}, \mathcal{F}'\rangle$ is the collection of hedgelets $\{\mathcal{F}(\mathbf{T})\}_{\mathbf{T}\in\mathcal{C}(\mathcal{F}\setminus\mathcal{F}')}$ where each hedgelet $\mathcal{F}(\mathbf{T})$ is a subgraph of $\mathcal{F}$ made of (i) $\mathcal{F}[\mathbf{V}(\mathcal{F}') \cup \mathbf{T}]$ and (ii) $\mathcal{F}[De(\mathbf{T})_{\mathcal{F}}]$ without bidirected edges.

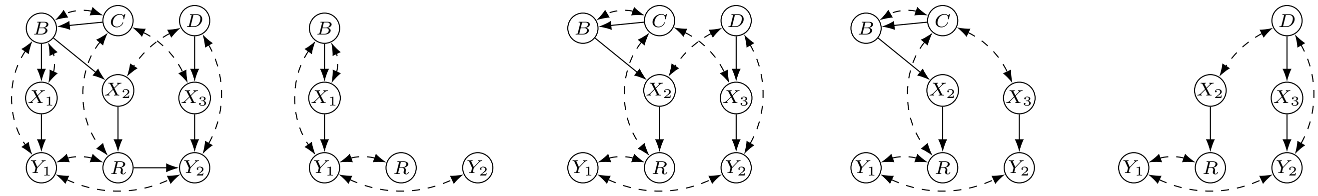
An s-thicket is a superimposition of hedges sharing a common root-set, where each hedge is also a superimposition of hedgelets. Intuitively speaking, if we encounter an s-thicket $\mathcal{T}$ for $P^*_{\mathbf{x}'}(\mathbf{y}')$ in $\mathcal{G}$, g-transporting $P^*_{\mathbf{x}'}(\mathbf{r})$, where $\mathbf{X}' = \mathbf{X} \cap \mathcal{T}$, is hindered because every existing experimental distribution either (a) exhibits discrepancies, (b) is based on an intervention on the variables we wish to measure, or (c) is not sufficient to pinpoint $P^*_{\mathbf{x}'}(\mathbf{r})$. Further, $P^*_{\mathbf{x}}(\mathbf{y})$ is not g-transportable since the negative result for $P^*_{\mathbf{x}'}(\mathbf{r})$ can be mapped to that for $P^*_{\mathbf{x}'}(\mathbf{y}')$ where $\mathbf{Y}' \subseteq \mathbf{Y}$ and $\mathbf{R} \subseteq An(\mathbf{Y}')_{\mathcal{G}\setminus\mathbf{X}}$.

Consider, for example, the causal graph $\mathcal{G}$ in Fig. 2a where $\boldsymbol{\Delta} = \{\emptyset, \{B\}\}$ and $\mathbb{Z} = \{\{\{C\}\}, \{\{X_1\}, \{X_3, R\}\}\}$. $\mathcal{G}$ without $R \to Y_2$ is an s-thicket for $P^*_{\mathbf{x}}(\mathbf{y})$ with respect to $\langle \mathcal{G}^{\boldsymbol{\Delta}}, \mathbb{Z}\rangle$. First, an experiment on $\{X_3, R\}$ matches (b) in Def. 4. Since the other two experiments do not match (a) nor (b) in Def. 4, there should be two hedges which do not intersect with C and X_1, respectively (Fig. 2b and Fig. 2c). The former, which disjoints with $\{C\}$, is also its only hedgelet. The latter, which does not contain $\{X_1\}$, is composed of two hedgelets based on the C-component decomposition of its top (i.e., the subgraph induced by removing its root-set) $\mathcal{C}(\mathcal{F}_b[\{B, C, D, X_2, X_3\}]) = \{\{B, C, X_3\}, \{D, X_2\}\}$. Now, we formally establish a connection between an s-thicket and the non-g-transportability of a query:

Lemma 3. *With respect to $\mathcal{G}^{\boldsymbol{\Delta}}$ and $\mathbb{Z}$, a causal effect $P^*_{\mathbf{x}}(\mathbf{y})$ is not g-transportable if there exists an s-thicket $\mathcal{T}$ formed for the causal effect.*

Proof sketch. Treating multiple domains as if they are *homogeneous*, the existence of $\mathcal{T}$ entails the existence of two models witnessing the non-g-transportability of $P^*_{\mathbf{x}'}(\mathbf{r})$, for some $\mathbf{X}' \subseteq \mathbf{X}$, from $\mathcal{G}^{\{\emptyset\}}$ and $\{\bigcup_i \mathbb{Z}^i\}$ (Lee, Correa, and Bareinboim 2019). However, the same models will not necessarily agree on some of distributions available in source domains. We incorporate selection variables into the parametrization to make the two models agree on $\mathbb{P}^{\Pi}_{\mathbb{Z}}$ while still disagreeing on $P^*_{\mathbf{x}'}(\mathbf{r})$. The parametrization (Lee, Correa, and Bareinboim 2019) is designed to produce the same distributions for the two models if at least one $R \in \mathbf{R}$ becomes independent to the UCs among $\mathbf{R}$, which isn't the case for $do(\mathbf{x})$. We modify each function for $R \in \mathbf{R}$ to return 0 when $S_R \neq 1$.[3] Consequently, the two models witness the non-g-transportability of $P^*_{\mathbf{x}'}(\mathbf{r})$, and the result will entail the same for $P^*_{\mathbf{x}'}(\mathbf{y}')$ in $\mathcal{T}'$, a graph where $\mathcal{T}$ is extended by adding directed paths from $\mathbf{R}$ to $\mathbf{Y}' \subseteq \mathbf{Y}$. □

[3] One can replace the constant 0 to an R-specific unobserved variables, which can be an (un)fair coin.



(a) $\mathcal{G}$ (b) $\mathcal{F}_a$ (c) $\mathcal{F}_b$ (d) $\mathcal{F}_b(\{B, C, X_3\})$ (e) $\mathcal{F}_b(\{D, X_2\})$

Figure 2: (a) A causal graph $\mathcal{G}$, which, without $R \rightarrow Y_2$, forms an s-thicket for $P^*_{\mathbf{x}}(\mathbf{y})$ given $\mathbf{\Delta} = \{\emptyset, \{B\}\}$ and $\mathbb{Z} = \{\{\{C\}\}, \{\{X_1\}, \{X_3, R\}\}\}$. The s-thicket is the superimposition of two hedges (b, c) where the latter further decomposed into two hedgelets (d, e).

At this point, the non-existence of an s-thicket is a necessary condition for the g-transportability of an unconditional causal effect. We will further show in Sec. 4 that this is sufficient too, which will be done by presenting an algorithm that returns a valid formula for the target effect whenever no s-thicket exists (Thm. 3). For the sake of a better presentation of the completeness of the graphical criterion for the conditional case in the next section, we put a corollary below based on Lemma 3 and Thm. 3:

Corollary 1. *With respect to $\mathcal{G}^{\mathbf{\Delta}}$ and $\mathbb{Z}$, a causal effect $P^*_{\mathbf{x}}(\mathbf{y})$ is not g-transportable if and only if there exists an s-thicket $\mathcal{T}$ formed for the causal effect.*

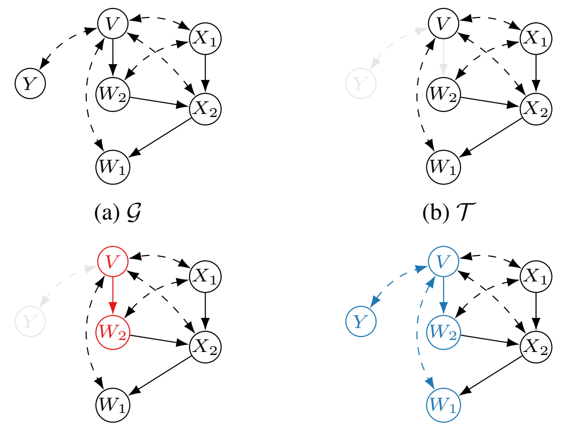
3.2 Non-g-transportability of a Conditional Interventional Distribution

We proceed to the graphical criterion for the g-transportation of $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$. We will assume that the query under consideration is *conditionally minimal* in the sense that there is no $W \in \mathbf{W}$ such that $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w}) = P^*_{\mathbf{x}\cup\{w\}}(\mathbf{y}|\mathbf{w} \setminus \{w\})$ by virtue of Rule 2 of *do*-calculus. Otherwise, we can repeatedly apply Rule 2 and obtain an equivalent minimal expression $P^*_{\mathbf{x},\mathbf{w}'}(\mathbf{y}|\mathbf{w} \setminus \mathbf{w}')$ (Cor. 1 (Shpitser and Pearl 2006a)). The conditional minimality is graphically translated to the existence of an active backdoor path from each of $W \in \mathbf{W}$ to some $Y \in \mathbf{Y}$ given $\mathbf{W} \setminus \{W\}$. We present a major theoretical result which authorizes the delegation of the characterization of a conditional causal effect to that of an unconditional one:

Theorem 1. *Let every $W \in \mathbf{W}$ have a backdoor path to $\mathbf{Y}$ in $\mathcal{G} \setminus \mathbf{X}$ active given $\mathbf{W} \setminus \{W\}$. A query $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is g-transportable if and only if $P^*_{\mathbf{x}}(\mathbf{y}, \mathbf{w})$ is g-transportable with respect to $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$.*

The sufficiency holds true since $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w}) = P^*_{\mathbf{x}}(\mathbf{y}, \mathbf{w}) / \sum_{\mathbf{y}} P^*_{\mathbf{x}}(\mathbf{y}, \mathbf{w})$. As for the necessity, suppose $P^*_{\mathbf{x}}(\mathbf{y}, \mathbf{w})$ is not g-transportable. If $P^*_{\mathbf{x}}(\mathbf{w})$ is g-transportable, then $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ must be non-g-transportable, otherwise a contradiction arises since $P^*_{\mathbf{x}}(\mathbf{y}, \mathbf{w})$ would be g-transportable as $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w}) P^*_{\mathbf{x}}(\mathbf{w})$. Then, it remains to prove that $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is not g-transportable whenever $P^*_{\mathbf{x}}(\mathbf{w})$ is not g-transportable with respect to $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$. Indeed, that is the case, as follows:

Theorem 2. *Let every $W \in \mathbf{W}$ have a backdoor path to $\mathbf{Y}$ in $\mathcal{G} \setminus \mathbf{X}$ active given $\mathbf{W} \setminus \{W\}$. A query $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is not g-transportable if $P^*_{\mathbf{x}}(\mathbf{w})$ is not g-transportable with respect to $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$.*



(a) $\mathcal{G}$ (b) $\mathcal{T}$ (c) $\mathcal{T}' = \mathcal{T} \cup \mathcal{E}$ (d) $\mathcal{T}'' = \mathcal{T} \cup \mathcal{E} \cup \mathcal{P}$

Figure 3: A causal diagram $\mathcal{G}$, and causal diagrams illustrating the phases of a non-g-transportability parametrization for $P^*_{\mathbf{x}}(y|\mathbf{w})$. (b) an s-thicket for $P^*_{\mathbf{x}}(\mathbf{w})$ given P^*, (c) the s-thicket with an extension in red, and (d) a path-witnessing subgraph (blue) augmented extended s-thicket.

Proof sketch. Let $\mathcal{T}'$ be a subgraph of $\mathcal{G}$ parametrized to demonstrate the non-g-transportability of $P^*_{\mathbf{x}'}(\mathbf{w}')$ given $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$ (Lemma 3). Pick some $W \in \mathbf{W}'$ that is also in the root-set of $\mathcal{T}'$, and fix a minimal subgraph $\mathcal{P} \subseteq \mathcal{G} \setminus \mathbf{X}$ witnessing an active backdoor path from W to some $Y \in \mathbf{Y}$ given $\mathbf{W} \setminus \{W\}$. $\mathcal{P}$ also includes any directed path from an active collider in $\mathcal{P}$ to its descendant in $\mathbf{W} \setminus \{W\}$. We construct two models for $\mathcal{T}' \cup \mathcal{P}$ while preserving the mechanisms in Lemma 3. We augment the exclusive-or-based parametrization for variables in $\mathcal{P}$ so that W and Y are correlated given $(\mathbf{W} \cap \mathcal{P}) \setminus \{W\}$. In the augmented models, the value of W is determined as the exclusive-or of two Ws computed in $\mathcal{T}'$ and in $\mathcal{P}$. The resultant models will disagree on $P^*_{\mathbf{x}'}(y|\mathbf{w}'')$ where $\mathbf{W}''$ is the subset of $\mathbf{W}$ in $\mathcal{T}' \cup \mathcal{P}$. Therefore, $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is not g-transportable with respect to $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$. $\square$

We provide an illustrative example in Fig. 3. For the sake of brevity, we assume a single domain setting with

Algorithm 1 GTR and GTRU, sound and complete g-transportability algorithms.

1: **function** GTR$(\mathbf{y}, \mathbf{x}, \mathbf{w}, \mathcal{G}, \boldsymbol{\Delta})$
input: $\mathbf{y}$, $\mathbf{x}$, $\mathbf{w}$: values for a query $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$; $\mathcal{G}$: causal diagram; $\boldsymbol{\Delta}$: domain discrepancies.
output: an estimator computing $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$.

2: **if** $\exists_{W \in \mathbf{W}} (W \perp\!\!\!\perp \mathbf{Y} \mid \mathbf{W} \setminus \{W\})_{(\mathcal{G} \setminus \mathbf{X})_{\underline{W}}}$ **then**
return GTR$(\mathbf{y}, \mathbf{x} \cup \{w\}, \mathbf{w} \setminus \{w\}, \mathcal{G}, \boldsymbol{\Delta})$.

3: **else**
return $Q / \sum_{\mathbf{y}} Q$ **where** $Q \leftarrow$ GTRU$(\mathbf{y} \cup \mathbf{w}, \mathbf{x}, \mathcal{G}, \boldsymbol{\Delta})$.

4: **function** GTRU$(\mathbf{y}, \mathbf{x}, \mathcal{G}, \boldsymbol{\Delta})$
output: an estimator computing $P^*_{\mathbf{x}}(\mathbf{y})$.

5: **if** $\exists_{\mathbf{Z} \in \mathbb{Z}^i \in \mathbb{Z}} (\mathbf{X} = \mathbf{Z} \cap \mathbf{V}) \wedge (\mathbf{S}^i \perp\!\!\!\perp \mathbf{Y})_{\mathcal{G}^{\boldsymbol{\Delta}} \setminus \mathbf{X}}$ **then**
return $P^i_{\mathbf{z} \setminus \mathbf{V}, \mathbf{x}}(\mathbf{y})$.

6: **if** $(\mathbf{V}' \leftarrow \mathbf{V} \setminus An(\mathbf{Y})_{\mathcal{G}}) \neq \emptyset$ **then**
return GTRU$(\mathbf{y}, \mathbf{x} \setminus \mathbf{V}', \mathcal{G} \setminus \mathbf{V}', \{\Delta^i \setminus \mathbf{V}' \mid \Delta^i \in \boldsymbol{\Delta}\})$.

7: **if** $(\mathbf{V}' \leftarrow (\mathbf{V} \setminus \mathbf{X}) \setminus An(\mathbf{Y})_{\mathcal{G}_{\overline{\mathbf{X}}}}) \neq \emptyset$ **then**
return GTRU$(\mathbf{y}, \mathbf{x} \cup \mathbf{v}', \mathcal{G}, \boldsymbol{\Delta})$.

8: **if** $|\mathcal{C}(\mathcal{G} \setminus \mathbf{X})| > 1$ **then**
return $\sum_{\mathbf{v} \setminus (\mathbf{y} \cup \mathbf{x})} \prod_{\mathbf{C} \in \mathcal{C}(\mathcal{G} \setminus \mathbf{X})}$ GTRU$(\mathbf{c}, \mathbf{v} \setminus \mathbf{c}, \mathcal{G}, \boldsymbol{\Delta})$.

9: **for** $\pi^i \in \Pi$ such that $(\mathbf{S}^i \perp\!\!\!\perp \mathbf{Y})_{\mathcal{G}^{\boldsymbol{\Delta}} \setminus \mathbf{X}}$, **for** $\mathbf{Z} \in \mathbb{Z}^i$ such that $\mathbf{Z} \cap \mathbf{V} \subseteq \mathbf{X}$ **do**

10: **return** ID$(\mathbf{y}, \mathbf{x} \setminus \mathbf{Z}, P^i_{\mathbf{z} \setminus \mathbf{V}, \mathbf{x} \cap \mathbf{Z}}, \mathcal{G} \setminus (\mathbf{Z} \cap \mathbf{X}))$ **unless** FAIL is returned.

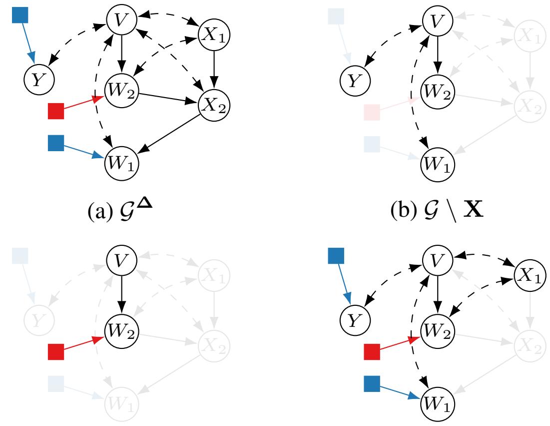
11: **throw** FAIL

P^* available. Given a causal graph $\mathcal{G}$ (Fig. 3a) and P^*, an s-thicket $\mathcal{T}$ is formed for $P^*_{\mathbf{x}}(\mathbf{w})$ (Fig. 3b). Two models are first constructed to disagree on $P^*_{\mathbf{x}}(v, w_1)$. Then, the result is mapped to $P^*_{\mathbf{x}}(\mathbf{w})$ via a graph $\mathcal{E}$ (red), resulting in a parametrization for $\mathcal{T}' = \mathcal{T} \cup \mathcal{E}$ (Fig. 3c). Pick $W_1 \in \mathbf{W}$, which is the only $\mathbf{W}$ in the root set of $\mathcal{T}'$, then find a backdoor path to Y given $\mathbf{W} \setminus \{W_1\}$. The path-witnessing subgraph $\mathcal{P} \in \mathcal{G}$ is shown in blue (Fig. 3d). A separate parametrization for $\mathcal{P}$ is merged with that for $\mathcal{T}'$ via an exclusive-or on W_1. Then, the two models disagree on $P^*_{\mathbf{x}}(y|\mathbf{w})$.

4 A Sound and Complete Algorithm for g-Transportability

In this section, we introduce GTR (Alg. 1), which is a sound and complete algorithm for solving any g-transportability instance, i.e., outputs an estimator for a given conditional interventional query $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ in a target domain with respect to $\langle \mathcal{G}^{\boldsymbol{\Delta}}, \mathbb{Z} \rangle$, when it exists. This algorithm smoothly and effectively combines the results underlying previous identification-transportability algorithms found in the literature, including (Tian 2002; Shpitser and Pearl 2006b; 2006a; Bareinboim and Pearl 2014; Lee, Correa, and Bareinboim 2019). The experiment specification $\mathbb{Z}$ and the corresponding distributions $\mathbb{P}^{\Pi}_{\mathbb{Z}}$ are defined globally, and do not change with the specific invocation of the algorithm. In contrast, variables $\mathbf{V}$ and selection variables $\mathbf{S}$ reflect graph $\mathcal{G}$ and discrepancies $\boldsymbol{\Delta}$, respectively, relative to the arguments passed to the current execution of the procedure.

We provide a line by line description where symbols such as $\mathcal{G}$, $\mathbf{V}$, $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{W}$ are to be interpreted relative to the



(a) $\mathcal{G}^{\boldsymbol{\Delta}}$ (b) $\mathcal{G} \setminus \mathbf{X}$ (c) $\mathcal{G}^{\boldsymbol{\Delta}}[An(W_2)_{\mathcal{G}}]$ (d) $\mathcal{G}^{\boldsymbol{\Delta}} \setminus \{X_2\}$

Figure 4: (a) A selection diagram $\mathcal{G}^{\boldsymbol{\Delta}}$ where $\boldsymbol{\Delta} = \{\emptyset, \{W_1, Y\}, \{W_2\}\}$ and $\mathbb{Z} = \{\emptyset, \{\emptyset\}, \{\{X_2\}\}\}$. (b,c,d) Graphs encountered during the execution of GTR to g-transport $P^*_{\mathbf{x}}(y|\mathbf{w})$.

current arguments of the algorithm. At Line 2, GTR, recursively transforms the given query using Rule 2 of *do*-calculus to guarantee it is conditionally minimal (and satisfies the requirement for Thm. 1). With this guarantee, the algorithm (Line 3) delegates the identification of the query, based on the definition of conditional probability, to GTRU, which handles unconditional queries. Overall, GTRU transforms the given unconditional query and divides the problem into the identification of (simpler) subqueries. Each subproblem is delegated to ID with a distribution $P^i_{\mathbf{z}}$ under some constraints on the domain π^i and the experiments on $\mathbf{Z} \in \mathbb{Z}^i$. Line 5, which is optional, checks whether an available distribution can be used to answer the query directly, i.e., $P^i_{\mathbf{z}}(\mathbf{y}) = P^*_{\mathbf{x}}(\mathbf{y})$, so as to return an estimator at an early stage. Line 6 narrows the scope of the problem by excluding variables that do not affect $\mathbf{Y}$ (Rule 3). Domain discrepancies are updated accordingly, since selection variables outside the scope have no effect on $\mathbf{Y}$. Line 7 maximizes the intervention set, which helps solving the problem, based on Rule 3. Line 8 breaks down the query into queries where $\mathbf{Y}$ in each subquery forms a C-component (Tian and Pearl 2002). Line 9 examines whether some experimental distribution $P^i_{\mathbf{z}} \in \mathbb{P}^{\Pi}_{\mathbb{Z}}$ can be used to identify the query. If valid, GTRU passes the query to ID with a slight modification of it and graph, taking into account the shared intervention between $\mathbf{Z}$ and $\mathbf{X}$. GTR runs in $O(v^4 z)$ where $v = |\mathbf{V}|$ and $z = \sum_i |\mathbb{Z}^i|$ (see Appendix A.4 for details).

We offer a running example regarding the identification of $P^*_{\mathbf{x}}(y|\mathbf{w})$ with a causal graph $\mathcal{G}$ (Fig. 3a), $\boldsymbol{\Delta} = \{\emptyset, \{W_1, Y\}, \{W_2\}\}$ (see $\mathcal{G}^{\boldsymbol{\Delta}}$ in Fig. 4a with $\mathbf{S}^2$ and $\mathbf{S}^3$ in blue and red), and $\mathbb{Z} = \{\emptyset, \{\emptyset\}, \{\{X_2\}\}\}$, i.e., the target domain has no distribution available while π^2 and π^3 provide an observational distribution and an experiment on X_2, respectively. Given a query $P^*_{\mathbf{x}}(y|\mathbf{w})$, GTR investigates whether there exists any $W \in \mathbf{W}$ that can be moved to the interventional part of the query. Fig. 4b shows $\mathcal{G} \setminus \mathbf{X}$

where the existence of a backdoor path between W and Y is figured out. Since $W_2 \leftarrow V \leftrightarrow Y$ and $W_1 \leftrightarrow V \leftrightarrow Y$ given W_2 as a descendant of the collider (V), it proceeds to identify $P^*_{\mathbf{x}}(y, \mathbf{w})$. GTRU attempts to refine the given graph with the ancestors of $\{Y, W_1, W_2\}$ (Line 6). Then, it checks whether the intervention $\{X_1, X_2\}$ is maximal. Next, it investigates the C-components of $\mathcal{G} \setminus \mathbf{X}$ (Fig. 4b). There are two C-components involving $\{W_2\}$ and $\{Y, V, W_1\}$. Hence, it factorizes the query to $P^*_{y,\mathbf{x},v,w_1}(w_2)$ and $P^*_{\mathbf{x},w_2}(y, v, w_1)$. The first query encounters Line 6 and it is refined, i.e., $P^*_{y,\mathbf{x},v,w_1}(w_2) = P^*_v(w_2)$ (Rule 3) with the graph in Fig. 4c. The query will reach Line 10 since $\{S_{W_2}\} \subseteq \mathbf{S}^{-2}$ (Lemma 1) and, eventually, ID identifies $P^*_v(w_2) = P^2(w_2|v)$, which corresponds to Rule 2. The second query passes conditions in Lines 5 to 9 since $(\{Y, V, W_1\} \perp\!\!\!\perp S_{W_2})$ in $\mathcal{G}^{\mathbf{\Delta}} \setminus \{X_2\}$ (Fig. 4d). Then, it makes use of $P^3_{x_2}$, since $\{X_2\} \subseteq \mathbf{X} \cup \{W_2\}$, to identify $P^*_{\mathbf{x},w_2}(y, v, w_1)$, which corresponds to identifying $Q^*_{x_1,w_2}(y, v, w_1)$ with $Q^3 = P^3_{x_2}$ in $\mathcal{G}^{\mathbf{\Delta}} \setminus \{X_2\}$ (Bareinboim and Pearl 2012a).

Theorem 3. GTRU *is sound and complete.*

Proof. (soundness) Let a subscript ℓ denote variables and values local to the function. The soundness of the algorithm is partially proved (Lee, Correa, and Bareinboim 2019) excluding the case where distributions from the heterogeneous source domains are utilized. It is sufficient to prove that $P^*_{\mathbf{x}_\ell}(\mathbf{y}_\ell) = P^i_{\mathbf{x}_\ell}(\mathbf{y}_\ell)$ for Lines 5 and 9 where the identification of $P^*_{\mathbf{x}_\ell}(\mathbf{y}_\ell)$ is delegated to that of $P^i_{\mathbf{x}_\ell}(\mathbf{y}_\ell)$ with $P^i_{\mathbf{z}}$ for some $\mathbf{Z} \in \mathbb{Z}^i$. By Lemma 1, $P^*_{\mathbf{x}_\ell}(\mathbf{y}_\ell) = P_{\mathbf{x}_\ell}(\mathbf{y}_\ell \mid \mathbf{S} = \mathbf{1})$. Since $(\mathbf{S}^i_\ell \perp\!\!\!\perp \mathbf{Y}_\ell)$ in $\mathcal{G}^{\mathbf{\Delta}_\ell}_\ell \setminus \mathbf{X}_\ell$ implies $(\mathbf{S}^i \perp\!\!\!\perp \mathbf{Y}_\ell)$ in $\mathcal{G}^{\mathbf{\Delta}} \setminus \mathbf{X}_\ell$, the equality $P_{\mathbf{x}_\ell}(\mathbf{y}_\ell \mid \mathbf{S} = \mathbf{1}) = P_{\mathbf{x}_\ell}(\mathbf{y}_\ell \mid \mathbf{S}^{-i} = \mathbf{1})$ holds true. Therefore, the soundness follows.

(completeness) We show that whenever GTRU fails to g-transport a given query $P^*_{\mathbf{x}}(\mathbf{y})$, there exists an s-thicket for the given query (Lemma 3). Given that GTRU imposes one more condition $(\mathbf{S}^i_\ell \perp\!\!\!\perp \mathbf{Y}_\ell)$ in $\mathcal{G}^{\mathbf{\Delta}_\ell}_\ell \setminus \mathbf{X}_\ell$ at Line 9 compared to GID, those qualified experiments $\mathbf{Z} \in \mathbb{Z}^i \in \mathbb{Z}$ can be considered as experiments conducted in the target domain so that the identification is reducible to GID given $\mathcal{G}$ with the qualified experiments (Lee, Correa, and Bareinboim 2019). Hence, when the algorithm fails to identify the query, there exists a thicket for $P^*_{\mathbf{x}}(\mathbf{y})$ (Thm. 3 (Lee, Correa, and Bareinboim 2019)). If every experiment $\mathbf{Z}$ satisfies items (b) and (c) in Def. 4, then the thicket is an s-thicket. Otherwise, we map the existence of a thicket $\mathcal{T}^\dagger$ to that of an s-thicket $\mathcal{T}$ — it remains to show $\Delta^i \cap \mathbf{R} \neq \emptyset$ (item (a) in Def. 4). First, there exists an $\mathbf{R}^\dagger$-rooted thicket $\mathcal{T}^\dagger \subseteq \mathcal{G}_\ell$ for $P^*_{\mathbf{x}_\ell}(\mathbf{y}_\ell)$, which is also for $P^*_{\mathbf{x}}(\mathbf{y})$. Since $\mathbf{R}^\dagger \subseteq An(\mathbf{Y}_\ell)_{\mathcal{G}_\ell \setminus \mathbf{X}_\ell} = \mathbf{V}_\ell \setminus \mathbf{X}_\ell$ and $\mathcal{G}_\ell[\mathbf{V}_\ell \setminus \mathbf{X}_\ell]$ is a C-component (Line 8), the thicket $\mathcal{T}^\dagger$ with its root set replaced with $\mathbf{V}_\ell \setminus \mathbf{X}_\ell$ is a valid thicket. Then, due to Prop. 1 (below), the modified thicket is an s-thicket for $P^*_{\mathbf{x}}(\mathbf{y})$ with respect to $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$. □

Proposition 1. $(\mathbf{S}^i \perp\!\!\!\perp \mathbf{Y})_{\mathcal{G}^{\mathbf{\Delta}} \setminus \mathbf{X}}$ *at Line 9 is equivalent to* $\mathbf{\Delta}^i \cap (\mathbf{V} \setminus \mathbf{X}) = \emptyset$.

Corollary 2. GTR *is sound and complete.*

Proof. The soundness of GTR follows from the soundness of GTRU (Thm. 3) and Rule 2. Its completeness follows from the completeness of GTRU (Thm. 3) and Thm. 1. □

Corollary 3. *The rules of do-calculus together with standard probability manipulations are complete for establishing g-transportability of conditional interventional distributions.*

Proof. This is due to: (i) Rule 2 of *do*-calculus and the definition of conditional probability under intervention for transitioning a conditional query to an unconditional one; and (ii) Rule 1 of *do*-calculus to determine whether to utilize the source domains (n.b. the selection variables as a condition as in Lemma 1 is implicit) along with the completeness of *do*-calculus with respect to GID. □

5 Conclusions

We investigated the challenge of learning conditional causal effects through the combination of observational and experimental distributions from heterogeneous domains, which unified many threads in the causal identifiability and transportability literature (Tian and Pearl 2002; Shpitser and Pearl 2006b; Huang and Valtorta 2006; Shpitser and Pearl 2006a; Bareinboim and Pearl 2013b; 2013a; 2012a; 2014; Lee, Correa, and Bareinboim 2019). This setting has been called g-transportability (Def. 3). Concretely, we developed a general treatment to the g-transportability problem in two ways. First, we introduced a complete graphical criterion, which leads to a novel parametrization strategy characterizing the g-transportability of any causal query (Lemma 3, Thm. 1, and Thm. 2). Second, we developed an efficient algorithm (GTR, Alg. 1, Thm. 3, and Cor. 2) that synthesizes heterogeneous datasets under the guidance of qualitative and transparent assumptions about the domain encoded as a causal graph. Further, we proved that Pearl's *do*-calculus is complete for this task (Cor. 3), which means that the inexistence of a derivation in this language implies that the intended causal explanation cannot be articulated based on the available evidence. We hope these new analytical tools can help lower the barrier for the broader research community to advance science through collaborative synthesis of shared datasets and knowledge.

A Appendices

A.1 Relationships among Identifiability and Transportability Problems

We illustrate in Fig. 5 the dimensions (problem space) of data-fusion tasks (Bareinboim and Pearl 2016), including identifiability, transportability, and its variants.

Specifically, the red axis (front–back) represents whether conditional queries can be answered as indicated by a suffix C. The blue axis (top–bottom) categorizes whether multiple heterogeneous domains are allowed, where ID (identifiability) is for a single domain, and TR (transportability) is for a pair of domains. The prefix M (multiple or meta) specifies that multiple source domains are available. Finally, the green axis (left–right) represents the expressiveness of available experiments. Without any prefixes (i.e., leftmost), the observational distribution P is available for the (target) domain, and any experiment is accessible for source domains (if available). Prefix Z- indicates that each domain is associated with experiments based on every subset of manipulable variables $\mathbf{Z} \subseteq \mathbf{V}$, while prefix G- implies that, for each domain, experiments can be conducted on an arbitrary combinations of subsets of $\mathbf{V}$. Our algorithm GTR would have been named as MGTRC if we followed this naming convention, which we decided to simplify.

A.2 Construction of a Counterexample for Non-g-transportability

We construct two models $\mathcal{M}_1$ and $\mathcal{M}_2$ demonstrating the non-g-transportability of a conditional interventional distribution in a g-transportability setting $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$. We first consider an s-thicket and its extension for an unconditional case. Next, we handle a conditional case with a backdoor path augmented to the extension. We fix the unconditional query of interest as $P^*_{\mathbf{x}}(\mathbf{w})$ and the conditional query is $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ where $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{W}$ satisfy the requirement in Thm. 1. The s-thicket for $P^*_{\mathbf{x}}(\mathbf{w})$ is denoted by $\mathcal{T}$ where its root set is $\mathbf{R}$. Let $\mathbf{X}' = \mathcal{T} \cap \mathbf{X}$. We let $\mathbf{U}_{\mathcal{T}}$ be UCs in $\mathcal{T}$. Let $\mathbb{H}$ be the hedgelets of s-thicket $\mathcal{T}$ and $\mathbb{H}(V) \subseteq \mathbb{H}$ be hedgelets containing $V \in \mathbf{V}(\mathcal{T})$.

Parametrizations for an s-Thicket This section provides parametrizations for two models corresponding to an s-thicket $\mathcal{T}$ demonstrating the non-g-transportability of $P^*_{\mathbf{x}'}(\mathbf{r})$, and, hence, that of $P^*_{\mathbf{x}}(\mathbf{r})$, as well. For both models, each unobserved confounder U in $\mathcal{T}$ except UCs among $\mathbf{R}$ consists of k-bits where k corresponds to the number of involved hedgelets. UCs among $\mathbf{R}$ has a single-bit (i.e., binary). Every bit of every unobserved confounder is an independent fair coin. We first define each function for $T \in \mathbf{V}(\mathcal{T}) \setminus \mathbf{R}$ where t is a $|\mathbb{H}(T)|$-bit integer. We use $v_{\mathcal{H}}$ and $u_{\mathcal{H}}$ to denote the bit of v and u corresponding to a hedgelet $\mathcal{H}$. For both models, the value of $T_{\mathcal{H}}$, the *bit* of T corresponding to a hedgelet $\mathcal{H} \in \mathbb{H}(T)$, is given by

$$t_{\mathcal{H}} \leftarrow \bigoplus_{V \in pa(T)_{\mathcal{H}}} v_{\mathcal{H}} \oplus \bigoplus_{U \in \mathbf{u}^T_{\mathcal{H}}} u_{\mathcal{H}}, \tag{2}$$

where $\mathbf{U}^T_{\mathcal{H}}$ is the set of UCs pointing towards T in $\mathcal{H}$. We define t as an integer corresponding to bits $\{t_{\mathcal{H}}\}_{\mathcal{H} \in \mathbb{H}(T)}$. Hedgelets parametrized this way satisfy an important property: the parity of the hedgelet-specific bits of the variables in the frontier (the parents of $\mathbf{R}$ in the hedgelet) is equal to the parity of the hedgelet-specific bits of all the UCs in the hedgelet such that all UCs, but for the crossing one (a UC connecting $\mathbf{R}$ and non-$\mathbf{R}$), are accounted twice. Hence, the bit-parity of the frontiers with respect to a hedgelet will be the same as the value of the crossing UC.

For example, consider the s-thicket (Fig. 6a) which consists of two hedgelets Fig. 6b and Fig. 6c. Red bidirected edges represent crossing UCs which are not canceled out through carrying bit-parity of red and blue UCs to $\mathbf{R}$. By construction, there are two bits for X_2 corresponding to each of hedgelets, which is shown in Fig. 6d.

Now, we parametrize variables in $\mathbf{R}$. First, pick any $R \in \mathbf{R}$, and label it with $R^\star$, for which the two models will equip distinctive parametrizations. We denote by $\mathbf{U}'_{\mathcal{T}}$ the UCs among $\mathbf{R}$, and $\mathbf{U}_{\mathcal{T}}$ or $\mathbf{U}'_{\mathcal{T}}$ with a variable as a superscript represents UCs pointing toward that variable within the scope. For $\mathcal{M}_2$, if $R = R^\star$,

$$r \leftarrow \Big(\bigwedge_{T \in pa(R)_{\mathcal{T}}} \mathbf{1}_{t=0} \wedge \bigwedge_{U \in \mathbf{U}^R_{\mathcal{T}} \setminus \mathbf{U}'^R_{\mathcal{T}}} u \wedge \mathbf{1}_{S_R \notin \mathbf{S} \vee S_R = *} \Big) \wedge \overline{\bigoplus \mathbf{u}'^R_{\mathcal{T}}}. \tag{3}$$

Otherwise,

$$r \leftarrow \Big(\bigwedge_{T \in pa(R)_{\mathcal{T}}} \mathbf{1}_{t=0} \wedge \bigwedge_{U \in \mathbf{U}^R_{\mathcal{T}} \setminus \mathbf{U}'^R_{\mathcal{T}}} u \wedge \mathbf{1}_{S_R \notin \mathbf{S} \vee S_R = *} \Big) \wedge \bigoplus \mathbf{u}'^R_{\mathcal{T}}. \tag{4}$$

Compared to the parametrization for a thicket, that for an s-thicket takes into account selection variables that point towards $\mathbf{R}$. The difference, $\mathbf{1}_{S_R \notin \mathbf{S} \vee S_R = *}$, makes the expression inside the parentheses in Eqs. (3) and (4) to be 0 for each source domain that exhibits discrepancy on the mechanism for R against the target domain, i.e., $\{\pi^i \mid R \in \Delta^i\}$.[4]

Proposition 2. *The two models $\mathcal{M}_1$ and $\mathcal{M}_2$ agree on $\mathbb{P}^{\Pi}_{\mathbb{Z}}$ and disagree on $P^*_{\mathbf{x}'}(\mathbf{r})$.*

Proof. For each domain $\pi^i \in \Pi$ where $\Delta^i \cap \mathbf{R} = \emptyset$, the above parametrization guarantees the agreement between the two models for $P^i_{\mathbf{z}}$ for every $\mathbf{Z} \in \mathbb{Z}^i$ since the parametrization is reduced to that for the problem in (Lee, Correa, and Bareinboim 2019). Hence, the two models disagree on $P^*_{\mathbf{x}'}(\mathbf{r})$ (more precisely speaking, under $do(\mathbf{x}' = 0)$, there exists non-zero probability where the two models $\mathbf{r}$'s bit-parities differ). To show their agreement on $P^i_{\mathbf{z}}$ when $\Delta^i \cap \mathbf{R} \neq \emptyset$, we utilize Cor. 1 (Lee, Correa, and Bareinboim 2019) where at least one $R \in \mathbf{R}$ becomes independent to $\mathbf{U}'^R_{\mathcal{T}}$ under $do(\mathbf{z})$. This guarantees that the two models agree on $P^i_{\mathbf{z}}$ since $\mathbf{1}_{S_R \notin \mathbf{S} \vee S_R = *}$ in Eq. (4) ensures that at least one R is suppressed to 0 or fixed to a constant if $R \in \mathbf{Z}$. Hence, the result follows. □

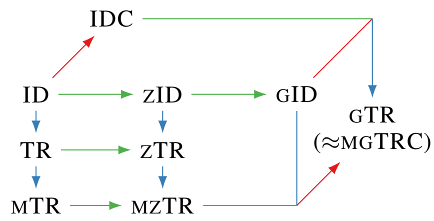


Figure 5: Generalized problems of identifiability and transportability laid out in 3D space. An edge between two problems represent predecessor-successor relationship.

[4]If suppressing a variable to a constant is less desirable, one might employ an R-specific unobserved variables, e.g., U_R, which can be a fair or unfair coin, where $R \leftarrow U_R$ if $S_R \neq *$.

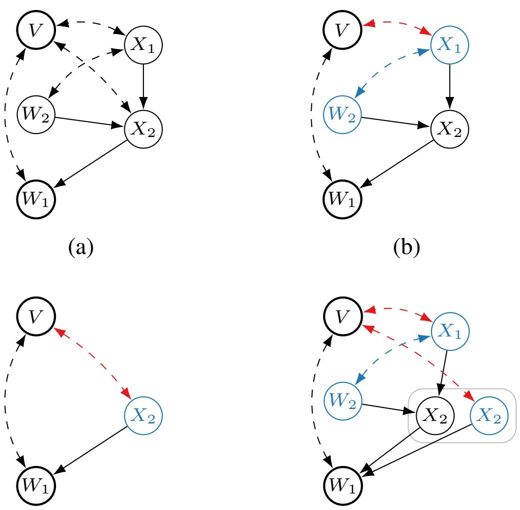


Figure 6: (a) an s-thicket (also a hedge) with $\{V, W_1\}$ as the root set (thick). (b,c) two hedgelets of the hedge with two C-components of $\mathcal{T} \setminus \mathbf{R}$ (blue) where red bidirected edges represent crossing UCs. (d) visualization of the parameterization of the s-thicket where X_2, which appears in both hedgelets among non-$\mathbf{R}$, consists of two bits.

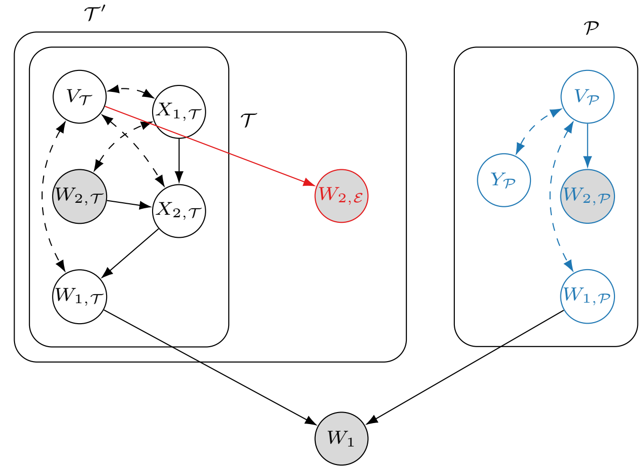


Figure 7: An illustration of parametrization for non-g-transportability of $P^*_{\mathbf{x}}(y|\mathbf{w})$. Each vertex represents a bit of corresponding variable except $W_{1,\mathcal{T}}$ and $W_{1,\mathcal{P}}$. For example, W_2 consists of three bits $W_{2,\mathcal{T}}$, $W_{2,\mathcal{E}}$, and $W_{2,\mathcal{P}}$. W_1 is the exclusive-or of $W_{1,\mathcal{T}}$ and $W_{1,\mathcal{P}}$. ($X_{2,\mathcal{T}}$ is of 2-bit as shown in Fig. 6d)

Parametrizations for an Extended s-Thicket Now we extend the s-thicket to demonstrate the non-g-transportability of $P^*_{\mathbf{x}}(\mathbf{w})$ utilizing the parametrizations above. To do so, we augment $\mathcal{T}$ with directed paths from $\mathbf{R}$ to some $\mathbf{W}' \subseteq \mathbf{W}$, which will be defined soon. By definition of s-thicket, $\mathbf{R} \subseteq An(\mathbf{W})_{\mathcal{G}\setminus\mathbf{X}}$, hence, for every $R \in \mathbf{R}$ a directed path from R to $\mathbf{W}$, not passing through $\mathbf{X}$, must exists. Among many possible directed paths from $\mathbf{R}$ to $\mathbf{W}$, we focus on directed paths that intersect $\mathbf{W}$ only at the endpoint. Let $\mathcal{G}'$ be $(\mathcal{G} \setminus \mathbf{X})_{\underline{\mathbf{W}}}$ without UCs. Then, let $\mathcal{E}$ (extender) be a minimal (possibly empty) subgraph of $\mathcal{G}'$ satisfying $\mathbf{R} \setminus \mathbf{W} \subseteq An(\mathbf{W} \cap \mathcal{E})_{\mathcal{E}}$ and let $\mathbf{W}'$ be the union of $\mathbf{R} \cap \mathbf{W}$ and the root set of $\mathcal{E}$. Intuitively, $\mathcal{E}$ represents how $\mathbf{R} \setminus \mathbf{W}$ is mapped to $\mathbf{W}' \setminus (\mathbf{W} \cap \mathbf{R})$. Let $\mathcal{T}' = \mathcal{T} \cup \mathcal{E}$.

We now construct two models for $\mathcal{T}'$ building on top of the parametrizations for $\mathcal{T}$ described before. Let $v_{\mathcal{T}}$ be the value of V in $\mathcal{T}$ based on the parametrization for the s-thicket. We define auxiliary variables $V_{\mathcal{E}}$ for each $V \in \mathbf{V}(\mathcal{E})$ as

$$v_{\mathcal{E}} \leftarrow \begin{cases} v_{\mathcal{T}} \oplus \bigoplus_{V' \in pa(V)_{\mathcal{E}}} v'_{\mathcal{E}} & \text{if } V \in \mathbf{R}, \\ \bigoplus_{V' \in pa(V)_{\mathcal{E}}} v'_{\mathcal{E}}. & \text{otherwise} \end{cases} \tag{5}$$

Then, we combine $v_{\mathcal{T}}$ and $v_{\mathcal{E}}$ to parametrize variables in $\mathcal{T}'$:

$$v \leftarrow \begin{cases} v_{\mathcal{T}} & \text{if } V \notin \mathcal{E}, \\ v_{\mathcal{E}} & \text{if } V \notin \mathcal{T}, \\ v_{\mathcal{E}} & \text{else if } V \in \mathbf{R}, \\ v_{\mathcal{T}} \mid\mid v_{\mathcal{E}} & \text{otherwise;} \end{cases} \tag{6}$$

where '||' concatenates two integers.

See Fig. 3 for an example, we map from $\mathbf{R} = \{V, W_1\}$ to $\mathbf{W} = \{W_1, W_2\}$. Since W_1 is shared (i.e., $W_1 \in \mathbf{R} \cap \mathbf{W}$), we only need to map from $\{V\}$ (i.e., $\mathbf{R} \setminus \mathbf{W}$) to $\{W_1, W_2\}$. In $\mathcal{G} \setminus \mathbf{X}$, there exists a directed path $V \rightarrow W_2$, which becomes $\mathcal{E}$. Since V has no parent in $\mathcal{E}$ and $V \in \mathbf{R}$, the parametrization of V is the same as in the previous section. W_2, which appears on both $\mathcal{T}$ and $\mathcal{E}$, consists of two bits where one bit comes from the previous parametrization and the other bit corresponds to the bit-parity of parents in $\mathcal{E}$, which is simply V.

Proposition 3. *The two models $\mathcal{M}_1$ and $\mathcal{M}_2$ agree on $\mathbb{P}^{\Pi}_{\mathbb{Z}}$ and disagree on $P^*_{\mathbf{x}'}(\mathbf{w}')$.*

Proof. Eq. (5) corresponds to carrying the bit-parity of $\mathbf{R} \setminus \mathbf{W}$ and combining the bit-parity onto a subset of $\mathbf{R} \cap \mathbf{W}$ if directed paths end up there. Any variables not involving $\mathcal{E}$ follow the parametrizations in the previous section. Otherwise, the bit-parity is transmitted through a separate bit without interfering the parametrization for the s-thicket.

If every $W \in \mathbf{W}'$ is not on the top of the s-thicket $\mathcal{T}$, $\mathbf{W}'$ are all binaries, and the two models will disagree on the distribution of the bit-parity of $\mathbf{w}'$ under $do(\mathbf{x} = 0)$. It is probable that a subset of $\mathbf{W}'$ resides on the top of $\mathcal{T}$, which implies that there are bits in those variables in addition to the bit-parity of $\mathbf{R}$. Since the two models agreed on $\mathbf{W}' \cap (\mathcal{T} \setminus \mathbf{R})$ in the parametrization for the s-thicket (i.e., where those extra bits are coming from), the disagreement on $P^*_{\mathbf{x}}(\mathbf{w}')$ follows. $\square$

Parametrizations for an Extended s-Thicket with a Path-Witnessing Subgraph We show a parametrization for the two models demonstrating the non-g-transportability of $P^*_{\mathbf{x}'}(y|\mathbf{w}'')$ for some $Y \in \mathbf{Y}$, and hence that of $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ as well (we will define $\mathbf{W}'' \subseteq \mathbf{W}$ later in this section). We start by constructing a graph which consists of the extension $\mathcal{T}'$ and a subgraph of $\mathcal{G}$ witnessing a backdoor path.

Choose $W \in \mathbf{W}'$ that is in the root set of $\mathcal{T}'$.[5] There must be one otherwise it contradicts the acyclicity of the graph since the root set cannot be empty. Then, we obtain in $\mathcal{G} \setminus \mathbf{X}$ a minimal subgraph $\mathcal{P}$, which witnesses a backdoor path between W and some $Y \in \mathbf{Y}$, i.e., $(W \not\perp\!\!\!\perp Y \mid (\mathbf{W} \cap \mathcal{P}) \setminus \{W\})_{\mathcal{P}}$. Note that the subgraph may not be only the path itself, but it might also include a subset of $\mathbf{W} \setminus \{W\}$ as the descendants of colliders in the path. For instance see the right side of Fig. 7, the *d*-connection path from Y to W_1 is $Y \leftrightarrow V \leftrightarrow W_1$ *given* W_2. Hence, the graph witnessing the path should include a directed path from V to W_2.

We parametrize $\mathcal{T}'' = \mathcal{T}' \cup \mathcal{P}$ building on top of the construction for $\mathcal{T}'$. Let $\{V_{\mathcal{P}} \mid V \in \mathcal{P}\}$ be auxiliary variables. Together with UCs in $\mathcal{P}$, we also add additional variable-specific unobserved variables in our models for those $V \in \mathcal{P}$ having no parent (either unobserved or observed). Let every $U_{\mathcal{P}} \in \mathbf{U}_{\mathcal{P}}$ be a fair coin. The auxiliary variables are parametrized as,

$$v_{\mathcal{P}} \leftarrow \bigoplus_{V' \in pa(V)_{\mathcal{P}}} v'_{\mathcal{P}} \oplus \bigoplus_{U_{\mathcal{P}} \in \mathbf{U}_{\mathcal{P}}} u_{\mathcal{P}}. \tag{7}$$

Recall that Y and W are d-connected in $\mathcal{P}$ given $(\mathbf{W} \setminus \{W\}) \cap \mathcal{P}$. Defining the values associated with $\mathcal{P}$ this way, whenever the $\mathcal{P}$-specific values of all (descendants of) colliders in $\mathcal{P}$ (i.e., $(\mathbf{W} \setminus \{W\}) \cap \mathcal{P}$) are observed to be 0, it is always the case that $Y_{\mathcal{P}} = W_{\mathcal{P}}$. Now, we combine constructions for $\mathcal{T}'$ and for $\mathcal{P}$ regarding $\mathcal{T}''$:

$$v \leftarrow \begin{cases} v_{\mathcal{T}'} \oplus v_{\mathcal{P}} & \text{if } V = W, \\ v_{\mathcal{T}'} & \text{if } V \notin \mathcal{P}, \\ v_{\mathcal{P}} & \text{else if } V \notin \mathcal{T}', \\ v_{\mathcal{T}'} || v_{\mathcal{P}} & \text{otherwise.} \end{cases} \tag{8}$$

This is nothing but the superimposition of the two parametrizations, while taking exclusive-or for W. Let $\mathbf{W}''$ be $\mathbf{W} \cap \mathcal{T}''$.

Proposition 4. *The two models $\mathcal{M}_1$ and $\mathcal{M}_2$ agree on $\mathbb{P}^{\Pi}_{\mathbb{Z}}$ and disagree on $P^*_{\mathbf{x}'}(y|\mathbf{w}'')$.*

Proof. Let a variable with a subscript $\mathcal{P}$ or $\mathcal{T}'$ be the bits of the variable corresponding to $\mathcal{P}$ or $\mathcal{T}'$, respectively. Let $\mathbf{W}''_{\mathcal{T}'} = \{W_{\mathcal{T}'} \mid W \in \mathcal{T}' \cap \mathbf{W}''\}$ and define $\mathbf{W}''_{\mathcal{P}'}$ similarly. We focus on a situation with $y_{\mathcal{P}} = 0$, $\mathbf{w}''_{\mathcal{P}} \setminus \{w_{\mathcal{P}}\} = 0$, and $w = 0$.

First, we rewrite the joint probability over $Y_{\mathcal{P}}$ and $\mathbf{W}''$ as:

$$\begin{aligned} &P^*_{\mathbf{x}'}(y_{\mathcal{P}}, \mathbf{w}'') \\ &= P^*_{\mathbf{x}'}(y_{\mathcal{P}}, \mathbf{w}'' \setminus \{w\}, w) \\ &= P^*_{\mathbf{x}'}(y_{\mathcal{P}}, \mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, \mathbf{w}''_{\mathcal{P}} \setminus \{w_{\mathcal{P}}\}, w) \\ &= P^*_{\mathbf{x}'}(y_{\mathcal{P}}, \mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, \mathbf{w}''_{\mathcal{P}} \setminus \{w_{\mathcal{P}}\}, w_{\mathcal{P}} \oplus w_{\mathcal{T}'}) \\ &= \sum_{w_{\mathcal{P}}, w_{\mathcal{T}'} | w_{\mathcal{P}} \oplus w_{\mathcal{T}'} = w} P^*_{\mathbf{x}'}(y_{\mathcal{P}}, \mathbf{w}''_{\mathcal{P}} \setminus \{w_{\mathcal{P}}\}, w_{\mathcal{P}}) \\ &\qquad\qquad P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, w_{\mathcal{T}'}). \end{aligned}$$

[5]It is probable to provide a parametrization for any $W \in \mathbf{W}'$. Restricting it to the root set results in a parametrization easy to interpret since the variables in the root set are binary while others can be of multi-bits.

Now observe that

$$\begin{aligned} &P^*_{\mathbf{x}'}(Y_{\mathcal{P}} = 0, \mathbf{W}''_{\mathcal{P}} \setminus \{W_{\mathcal{P}}\} = 0, W_{\mathcal{P}} = 1) = 0 \text{ and} \\ &P^*_{\mathbf{x}'}(Y_{\mathcal{P}} = 1, \mathbf{W}''_{\mathcal{P}} \setminus \{W_{\mathcal{P}}\} = 0, W_{\mathcal{P}} = 0) = 0, \end{aligned}$$

since $Y_{\mathcal{P}} = W_{\mathcal{P}}$ whenever $\mathbf{W}''_{\mathcal{P}} \setminus \{W_{\mathcal{P}}\} = 0$. Therefore,

$$P^*_{\mathbf{x}'}(y_{\mathcal{P}}, \mathbf{w}'') = P^*_{\mathbf{x}'}(y_{\mathcal{P}}, \mathbf{w}''_{\mathcal{P}}) P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, w_{\mathcal{T}'})$$

when $\mathbf{w}''_{\mathcal{P}} \setminus \{w_{\mathcal{P}}\} = 0$ and $w = 0$ where $w_{\mathcal{T}'} = y_{\mathcal{P}}$. Then, the probability $P^*_{\mathbf{x}'}(y_{\mathcal{P}}|\mathbf{w}'')$ can be written as

$$\begin{aligned} &\Big(P^*_{\mathbf{x}'}(y_{\mathcal{P}}, \mathbf{w}_{\mathcal{P}}) P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, w_{\mathcal{T}'})\Big) / \\ &\quad \Big(\sum_{y'_{\mathcal{P}} \in \{0,1\}} P^*_{\mathbf{x}'}(y'_{\mathcal{P}}, \mathbf{w}''_{\mathcal{P}} \setminus \{w_{\mathcal{P}}\}, W_{\mathcal{P}} = y'_{\mathcal{P}}) \cdot \\ &\qquad\qquad P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, W_{\mathcal{T}'} = y'_{\mathcal{P}}) \Big). \end{aligned}$$

Two models agree on the divisor because

1. When $\mathbf{W}''_{\mathcal{P}} \setminus \{W_{\mathcal{P}}\} = 0$, $Y_{\mathcal{P}} = W_{\mathcal{P}} = a$ only if every $U_{\mathcal{P}} \in \mathbf{U}_{\mathcal{P}}$ is equal to a, where $a \in \{0, 1\}$. Since every $U_{\mathcal{P}}$ is a fair coin, it follows that $P^*_{\mathbf{x}'}(Y_{\mathcal{P}} = 0, \mathbf{w}''_{\mathcal{P}} \setminus \{w_{\mathcal{P}}\}, W_{\mathcal{P}} = 0) = P^*_{\mathbf{x}'}(Y_{\mathcal{P}} = 1, \mathbf{w}''_{\mathcal{P}} \setminus \{w_{\mathcal{P}}\}, W_{\mathcal{P}} = 1)$. Let this probability be denoted by C for the next item.
2. **Claim**: The two models agree on $\sum_{y'_{\mathcal{P}} \in \{0,1\}} C \cdot P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, w_{\mathcal{T}'} = y'_{\mathcal{P}})$ (see below).

By construction, the two models share the same parametrization and agree on $P^*_{\mathbf{x}'}(y_{\mathcal{P}} = 0, \mathbf{w}_{\mathcal{P}} = 0)$ in the dividend. These entail that the two models will differ on the conditional interventional distribution if they disagree on $P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, W_{\mathcal{T}'} = 0)$. We investigate the value for $\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}$ so that the two models will disagree on $P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, W_{\mathcal{T}'} = 0)$. Split $\mathbf{W}''_{\mathcal{T}'}$ into two parts: i) those appearing in the top of the s-thicket $\mathcal{T}$ and ii) those not[6]. Obtain the value for the former which witnesses the disagreement of $P^*_{\mathbf{x}'=0}(\mathbf{w}' = 0)$ in the extended thicket (i.e., the joint distribution of the former and $\mathbf{w}' = 0$ is positive under $do(\mathbf{x}' = 0)$ for each model). Fix the value for the latter to 0. Then, the two models disagree on $P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, W_{\mathcal{T}'} = 0)$. This concludes the proof. ☐

Claim. *The two models agree on $\sum_{y'_{\mathcal{P}} \in \{0,1\}} C \cdot P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, w_{\mathcal{T}'} = y'_{\mathcal{P}})$.*

Proof. Here C does not depend on $Y_{\mathcal{P}}$ and can come out of the sum. Let $\mathbf{A}$ be a set of indicator variables, such that $A_R \in \mathbf{A}$ corresponds to the event that for $R \in \mathbf{R}$, the term in parentheses in Eqs. (3) and (4) is equal to 1. Then, we split $\mathbf{W}''_{\mathcal{T}'}$ into two parts based on whether $W \in \mathbf{W}''_{\mathcal{T}'}$ is in the top of the s-thicket $\mathcal{T}$. We use $\hat{\mathbf{W}}''_{\mathcal{T}'}$ for those in the top and $\bar{\mathbf{W}}''_{\mathcal{T}'}$ for the rest, which includes $W_{\mathcal{T}'}$. Then,

$$\begin{aligned} P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'}) &= P^*_{\mathbf{x}'}(\bar{\mathbf{w}}''_{\mathcal{T}'} | \hat{\mathbf{w}}''_{\mathcal{T}'}) P^*_{\mathbf{x}'}(\hat{\mathbf{w}}''_{\mathcal{T}'}) \\ &= P^*_{\mathbf{x}'}(\bar{\mathbf{w}}''_{\mathcal{T}'}, \mathbf{a} = 1 | \hat{\mathbf{w}}''_{\mathcal{T}'}) P^*_{\mathbf{x}'}(\hat{\mathbf{w}}''_{\mathcal{T}'}) + \\ &\qquad P^*_{\mathbf{x}'}(\bar{\mathbf{w}}''_{\mathcal{T}'}, \mathbf{a} \neq 1 | \hat{\mathbf{w}}''_{\mathcal{T}'}) P^*_{\mathbf{x}'}(\hat{\mathbf{w}}''_{\mathcal{T}'}) \end{aligned}$$

[6]For example, $W_{2,\mathcal{T}}$ is the one appearing in the top of $\mathcal{T}$ while $W_{2,\mathcal{E}}$ and $W_{1,\mathcal{T}}$ are not.

By construction, the two models agree on $P^*_{\mathbf{x}'}(\bar{\mathbf{w}}''_{\mathcal{T}'}, \mathbf{a} \neq 1|\hat{\mathbf{w}}''_{\mathcal{T}'})$ and $P^*_{\mathbf{x}'}(\hat{\mathbf{w}}''_{\mathcal{T}'})$. They disagree only when $\mathbf{a} = 1$. In such a situation, each of the two models yields a discrete uniform distribution over $\bar{\mathbf{w}}''_{\mathcal{T}'}$ contingent to $\bigoplus \bar{\mathbf{w}}''_{\mathcal{T}'} = 0$ in one model and $\bigoplus \bar{\mathbf{w}}''_{\mathcal{T}'} = 1$ in the other model. Hence, $P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, w_{\mathcal{T}'} = 0)$ in one model is the same as $P^*_{\mathbf{x}'}(\mathbf{w}''_{\mathcal{T}'} \setminus \{w_{\mathcal{T}'}\}, w_{\mathcal{T}'} = 1)$ in other model (vice versa). Therefore, the claim holds. □

We provide the resulting parametrization for Fig. 3 in Table 1, showing only a subset of probabilities essential in understanding our strategy for building two models disagreeing on $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$.

A.3 Theorems and Proofs

Lemma 3. *With respect to $\mathcal{G}^{\mathbf{\Delta}}$ and $\mathbb{Z}$, a causal effect $P^*_{\mathbf{x}}(\mathbf{y})$ is not g-transportable if there exists an s-thicket $\mathcal{T}$ formed for the causal effect.*

Proof. This follows from Prop. 3. □

Theorem 2. *Let every $W \in \mathbf{W}$ have a backdoor path to $\mathbf{Y}$ in $\mathcal{G} \setminus \mathbf{X}$ active given $\mathbf{W} \setminus \{W\}$. A query $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is not g-transportable if $P^*_{\mathbf{x}}(\mathbf{w})$ is not g-transportable with respect to $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$.*

Proof. This follows from Prop. 4. □

Theorem 1. *Let every $W \in \mathbf{W}$ have a backdoor path to $\mathbf{Y}$ in $\mathcal{G} \setminus \mathbf{X}$ active given $\mathbf{W} \setminus \{W\}$. A query $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is g-transportable if and only if $P^*_{\mathbf{x}}(\mathbf{y}, \mathbf{w})$ is g-transportable with respect to $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$.*

Proof. Its sufficiency holds true due to the definition of conditional probability under intervention. For its necessity, we consider two cases depending on whether $P^*_{\mathbf{x}}(\mathbf{w})$ is g-transportable. Given $P^*_{\mathbf{x}}(\mathbf{y}, \mathbf{w})$ is not g-transportable, if $P^*_{\mathbf{x}}(\mathbf{w})$ is g-transportable, it is immediate that $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is not g-transportable. Given both $P^*_{\mathbf{x}}(\mathbf{w})$ and $P^*_{\mathbf{x}}(\mathbf{y}, \mathbf{w})$ are not g-transportable, non-g-transportability of $P^*_{\mathbf{x}}(\mathbf{y}|\mathbf{w})$ is is proved in Thm. 2. Hence, the result follows. □

Proposition 1. $(\mathbf{S}^i \perp\!\!\!\perp \mathbf{Y})_{\mathcal{G}^{\mathbf{\Delta}} \setminus \mathbf{X}}$ *at Line 9 is equivalent to* $\mathbf{\Delta}^i \cap (\mathbf{V} \setminus \mathbf{X}) = \emptyset$.

Proof. By the algorithm, every variable is an ancestor of $\mathbf{Y}$, i.e., $\mathbf{V} = An(\mathbf{Y})_{\mathcal{G}}$, and $\mathbf{V}$ is the union of two disjoint sets $\mathbf{X}$ and $An(\mathbf{Y})_{\mathcal{G} \setminus \mathbf{X}}$. That is, there exists a directed path from every variable in $\mathbf{V} \setminus \mathbf{X}$ to $\mathbf{Y}$. Hence, the test will be failed if and only if there exists a selection variable pointing $\mathbf{V} \setminus \mathbf{X}$, that is, $\Delta^i \cap (\mathbf{V} \setminus \mathbf{X}) = 0 \Leftrightarrow (\mathbf{S}^i \perp\!\!\!\perp \mathbf{Y})_{\mathcal{G}^{\mathbf{\Delta}} \setminus \mathbf{X}}$. □

A.4 Runtime Analysis of the Algorithms

We provide a runtime analysis of algorithms GTR and GTRU as a whole. Given $\langle \mathcal{G}^{\mathbf{\Delta}}, \mathbb{Z} \rangle$, let v and z, respectively, be the number of vertices in the causal diagram and the number of all experiments across studies. Without loss of generality, we assume that every source domain associates with, at least, one distribution, i.e., $|\mathbb{Z}^i| \geq 1$ so that $|\Pi| \leq z$.

GTR can be called recursively at most $|\mathbf{W}| \leq v$ times. GTRU will be invoked once by GTR. Conditions in Line 6, 7, and 8 will be satisfied only once until the given unconditional query is factorized. Each factorized query will, then, undergo 6, 7 once and arrive at Line 9. ID can be called at most the number of experiments z. ID, which is also a recursive algorithm, runs in $O(v^3)$. Hence, the number of function calls for GTR, GTRU, and ID is $O(v)$ (due to recursive application of Rule 2), $O(v)$ (due to factorization), and $O(vz)$ (factorization and qualified experiments), respectively, where each algorithm contains graphical operations (d-separation, graph manipulation, etc) that can be run in $O(v^2)$. Consequently, the algorithm runs in $O(v^4 z)$.

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v	w_1	$P^*_{\mathbf{x}}(v, w_1)$ $\mathcal{M}_1$	$\mathcal{M}_2$
0	0	0.5	0.375
	1	0.375	0.5
1	0	0	0.125
	1	0.125	0

(a) s-thicket

w_1	w_2	$P^*_{\mathbf{x}}(w_1, w_2)$ $\mathcal{M}_1$	$\mathcal{M}_2$
0	0	0.25	0.1875
	1	0.25	0.1875
	2	0	0.0625
	3	0	0.0625
1	0	0.1875	0.25
	1	0.1875	0.25
	2	0.0625	0
	3	0.0625	0

(b) extended s-thicket

w_1	w_2	y	$P^*_{\mathbf{x}}(w_1, w_2, y)$ $\mathcal{M}_1$	$\mathcal{M}_2$
0	0	0	0.0625	0.046875
		1	0.046875	0.0625
	⋮			

(c) augmented extended s-thicket (joint)

w_1	w_2	y	$P^*_{\mathbf{x}}(y \mid w_1, w_2)$ $\mathcal{M}_1$	$\mathcal{M}_2$
0	0	0	≈0.5714	≈0.4286
		1	≈0.4286	≈0.5714
	⋮			

(d) augmented extended s-thicket (conditional)

Table 1: Probability distributions of Fig. 3a corresponding to the parametrization of (a) an s-thicket, (b) an extended s-thicket, (c) a path-subgraph augmented extended s-thicket (as a joint distribution, unnecessary values are omitted), and (d) a path-subgraph augmented extended s-thicket (conditional distribution). W_2 in (c,d) can take values from 0 to 7 (three bits as shown in Fig. 7).

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